Comment on "Fluctuation-Driven First-Order Transition in Pauli-Limited d-Wave Superconductors":

Recently, the presence of a kind of Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state in a heavy fermion superconductor $CeCoIn_5$ at low temperatures in a field $\mathbf{H}$ parallel to the superconducting layers ($\mathbf{H} \perp c$) has been clarified through thermodynamic [1,2] and ultrasound [3] measurements. Dalidovich and Yang (DY) [4] have argued by assuming the Pauli-limiting, i.e., neglecting the presence of vortices, that a fluctuation-driven first order transition (FOT) resulting from their perturbative renormalization-group analysis be consistent with the apparent FOT near $H_{c2}(T)$ between the normal and FFLO states in $CeCoIn_5$. On the other hand, the observed anomaly in the sound velocity seems to be consistent only with a change of tilt elasticity of vortices due to a growth of a modulation parallel to $\mathbf{H}$ [3].

Below, we point out that the DY's neglect of vortices is never justified in addressing the phase diagram of $CeCoIn_5$. First, the anisotropy γ in coherence lengths of $CeCoIn_5$ is so small that the orbital depairing inducing the vortices is not negligible even in the mean field (MF) analysis. The γ-value estimated from H_{c2}-data [5] is at most 2.4, and hence, $CeCoIn_5$ is less anisotropic than $YBa_2Cu_3O_{6-\delta}$ with $\gamma \geq 5$ and $\xi_0/\gamma < s$, where ξ_0 and s are the in-plane coherence length and the spacing between neighboring layers, respectively. Both materials have quasi two-dimensional electronic states. However, the phase diagram of $YBa_2Cu_3O_{6-\delta}$ in $\mathbf{H} \perp c$ is explained as that of Josephson vortex (JV) states [6], implying the importance of interlayer couplings, while even an entire confinement of vortices in the interlayer spacings cannot occur in $CeCoIn_5$ with $\xi_0/\gamma > s$.

Next, DY assume the normal to FFLO transition at $H_{c2}(T)$ to be of second order in the MF approximation. The FOT-like behaviors near H_{c2} in $CeCoIn_5$ [7,1] are already visible far above the temperature $T_{\rm FFLO}$ below which $H_{c2}(T)$ is the MF boundary between the normal and FFLO states. In addition, a slight hysteresis [1] has appeared just *above* $T_{\rm FFLO}$ in sweeping the temperature and hence, has nothing to do with the presence of the FFLO state below $T_{\rm FFLO}$. Under the circumstances, assuming [4] a second order MF transition at $H_{c2}(T)$ *only* below $T_{\rm FFLO}$ is clearly unreasonable.

Once including the fluctuation, the neglect of the vortices becomes more serious. The melting of JV solid is a weak FOT, which easily becomes continuous due to a pinning disorder, and there is no phase coherence in the resulting JV liquid [6] below the (sharp) crossover line $H_{c2}(T)$. This picture on the vortex phase diagram is unaffected by the character of the MF transition at H_{c2} and a FFLO modulation which, due to the orbital depairing, tends to be formed along $\mathbf{H}$ [8]. In the Pauli-limited case, however, the noncritical FOT [4] should occur just *above* H_{c2} because it follows by assuming H_{c2} as a possible critical field, and the FFLO modulation direction is not fixed by $\mathbf{H}$. That is, since the fluctuation effect in the case with the orbital depairing is incompatible with that in the Pauli-limited case, assuming the Pauli-limiting is not valid even qualitatively in $CeCoIn_5$ showing the vortex elasticity below $T_{\rm FFLO}$ [3]. A slight hysteresis seen in $\mathbf{H} \perp c$ only at low temperatures and the FOT-like behavior in $CeCoIn_5$ have been explained within the approach including both the orbital and spin depairings [8].

Further, according to the thermal conductivity data [9,10], the normal to FFLO crossover at H_{c2} in an organic superconductor λ-$(BETS)_2GaCl_4$, where a convincing sign of a FFLO transition below H_{c2} was found [9], is clearly continuous. If the fluctuation of this material be much weaker than that of $CeCoIn_5$, this might not contradict the DY's FOT based on a continuous MF transition. However, judging from an experimental phase diagram [11] in $\mathbf{H} \parallel c$ which is similar to those of κ-$(ET)_2$-salts examined elsewhere [12], λ-$(BETS)_2GaCl_4$ has a much stronger fluctuation than $CeCoIn_5$ in spite of its γ-value ($\simeq 5$) [9] in magnitude close to that of $CeCoIn_5$. As indicated elsewhere [8], such a continuous normal to FFLO crossover [9] is naturally explained as a fluctuation effect once taking account of the orbital depairing.

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